4.2

The Password Is… Operations!

Arithmetic and Geometric Sequences

LEARNING GOALS

In this lesson, you will:

- Determine the next term in a sequence.
- Recognize arithmetic sequences.
- Determine the common difference.
- Recognize geometric sequences.
- Determine the common ratio.

KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

Nicknames and code names are often used for football plays, people—even our pets. But code names are no joke when it comes to important diplomats. In fact, the Secret Service assigns a code name to the president of the United States and the first family. Some classic code names for former U.S. presidents were "Tumbler" for President George Walker Bush, "Timberwolf" for President George Herbert Walker Bush, "Deacon" for President Jimmy Carter, and "Lancer" for President John Kennedy.

One of the most famous code names for a president was "Rawhide." In released Secret Service radio communications from the 1980s, the agents can be heard saying, "Rawhide is okay" after someone fired a gun at the president. However, it quickly became apparent that "Rawhide" was not okay and that he had in fact been shot during the assassination attempt. We now know that "Rawhide" was actually former President Ronald Reagan, who did survive the attempt on his life. Unfortunately, it was after that assassination attempt that codes names for presidents became public knowledge.

Presidents and other important diplomats still have code names, but since these have become public knowledge, they are more for tradition. You don't think that the Secret Service would let important things like code names become general knowledge, do you?

PROBLEM 1 What Comes Next, and How Do You Know?

1. Carefully cut out Sequences A through P. Make sure you do not cut away the letter representing the sequence.
2. Determine the unknown terms of each sequence. Describe the pattern under each sequence.
3. Sort the sequences into groups based on common characteristics. In the space provided, record the following information for each of your groups.
   - List the letters of the sequences in each group.
   - Provide a rationale as to why you created each group.

4. What mathematical operation(s) did you perform in order to determine the next terms of each sequence?

A

45, 90, 180, 360, _____ , _____ ,

_____ , . . .

B

−4, −2, 0, 2, _____ , _____ ,

_____ , . . .

C

−2, −6, −18, −54, _____ , _____ ,

_____ , . . .

D

2, 5, 10, 17, _____ , _____ ,

_____ , . . .

E

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}$, _____ , _____ ,

_____ , . . .

F

1234, 123.4, 12.34, 1.234, _____ ,

_____ , _____ , . . .

G

1, −2, 3, −4, 5, _____ , _____ ,

_____ , _____ , . . .

H

−20, −16, −12, −8, −4, _____ ,

_____ , _____ , . . .

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I

1, 10, 100, 1000, _____ , _____ , . . .

J

$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8}$, _____ , _____ , . . .

K

6.5, 5, 3.5, 2, _____ , _____ ,

_____ , . . .

L

86, 85, 83, 80, 76, _____ , _____ , . . .

M

$-16, 4, -1, \frac{1}{4}$, _____ , _____ , . . .

N

1473.2, 1452.7, 1432.2, 1411.7, _____,

_____ , _____ , . . .

O

$\sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1, 0, \sqrt{-1}$ _____ ,

_____ , . . .

P

−4, 12, −36, 108, _____ , _____ , . . .

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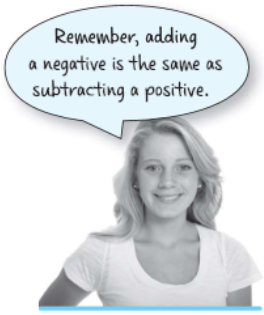
PROBLEM 2 Arithmetic, My Dear Watson!

You can describe a pattern as adding a constant to, or subtracting a constant from each term to determine the next term for some sequences. For other sequences, you can describe the pattern as multiplying or dividing each term by a constant to determine the next term. Still other sequences cannot be described either way.



An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a positive or negative constant is added to each term to produce the next term. This positive or negative constant is called the **common difference**. The common difference is typically represented by the variable *d*.

The common difference of a sequence is positive if the same *positive number* is added to each term to produce the next term. The common difference of a sequence is negative if the same *negative number* is added to each term to produce the next term.

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Consider the sequence shown.

11, 9, 7, 5, ...

The pattern is to add the same negative number, −2, to each term to determine the next term.

Sequence: 11 (add −2) 9 (add −2) 7 (add −2) 5 ...

This sequence is arithmetic and the common difference *d* is −2.

1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

   a. How would the pattern change?

   b. Is the sequence still arithmetic? Why or why not?

   c. If possible, write the first 5 terms of the new sequence.

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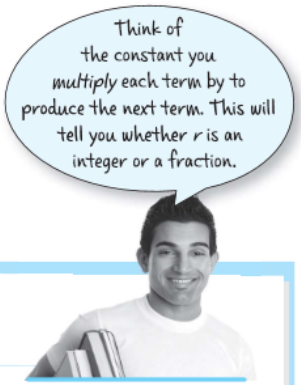
2. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?*

   a. List those sequences that are arithmetic.

   b. Write the common difference on each arithmetic sequence you cut out.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the **common ratio**. The common ratio is represented by the variable *r*.



Consider the sequence shown.

1, 2, 4, 8, . . .

The pattern is to multiply each term by the same number, 2, to determine the next term.

multiply by 2, multiply by 2, multiply by 2

Sequence: 1, 2, 4, 8, . . .

This sequence is geometric and the common ratio *r* is 2.

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3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.

a. How would the pattern change?

b. Is the sequence still geometric? Explain your reasoning.

c. If possible, write the first 5 terms for the new sequence.

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$.

   a. How would the pattern change?

   b. Is the sequence still geometric? Why or why not?

   c. If possible, write the first 6 terms for the new sequence.

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is -2.

   a. How would the pattern change?

   b. Is the sequence still geometric? Explain your reasoning.

   c. If possible, write the first 6 terms for the new sequence.

6. Consider the sequence shown.

   270, 90, 30, 10, . . .

   Devon says that he can determine each term of this sequence by multiplying each term by $\frac{1}{3}$, so the common ratio is $\frac{1}{3}$. Chase says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.

7. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know*? again.

   a. List those sequences that are geometric.

   b. Write the common ratio on each geometric sequence you cut out.

8. Consider the sequences from Problem 1 that are neither arithmetic nor geometric.

   a. List these sequences.

   b. Explain why these sequences are neither arithmetic nor geometric.

9. Consider the first two terms of a sequence.

3, 6, . . .

Dante says, "This is how I wrote the sequence for the given terms."

3, 6, 9, 12, . . .

Kira says, "This is the sequence I wrote."

3, 6, 12, 24, . . .

Who is correct? Explain your reasoning.

10. Using the terms given in Question 10, write a sequence that is neither arithmetic nor geometric. Then, have your partner tell you what the pattern is in your sequence.

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11. How many terms did your partner need before the pattern was recognized?

12. Consider the sequence 2, 2, 2, 2, 2. . . Identify the type of sequence it is and describe the pattern.

Be prepared to share your solutions and methods.